**Quantum Model (Drude)**

Diffusion, being a transport property, can be expected to evince the transition from delocalization to localization more transparently.

**Quantum Model: Diagrammatic Expansion**

A classical particle would be completely localized if its energy were less than the peak of any of the disordered wells. The quantum case is more complicated. A particle could be localized even if its energy were greater than any of the wells, if repeated reflections were to somehow constructively interfere to create a local standing wave. And yet a particle even with energy lower than many of those wells could be extended via tunneling (and in a perfectly ordered lattice, the wavefunctions *will* be extended). Still, it seems quantum particles are more easily localized than classical ones.

We can calculate the diffusion coefficient, with the usual Boltzman equation, but I haven’t done this. And we can also do this with GF. The one of interest would be the density-density correlation function, displayed below:



where ρ(r) = ψ†(r)ψ(r). It can be related to the diffusion ‘constant’, D(q,ω). The Fourier transform of the correlation function is related to the diffusion coefficient. Turns out (at T = 0 ?),



I don’t know how to prove this statement, but we can at least see that some of our prior results are consistent with it. In the EM folder, for instance, when working out the properties of metals, we found an expression for χirr(q,ω) = χ(q,ω) = e2ΠR(q,ω) [note that in so far as we’re presently dealing with non-interacting electrons, χirr(q,ω) = χ(q,ω)]:



In our typical units we have ε0 🡪 1, so:



We also found:



And we can relate these two:



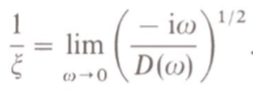
which is in accordance with the formula. Note that the Wolfle paper says that D(ω) and σ(ω) relate via D(0,ω)/D0 = σ(0,ω)/σ0, where the 0 presumably refers to ω = 0. The particle’s diffusive behavior is embedded in the poles of Π. Let’s consider the long time (small ω) limit.



Assuming this limit exists, then let’s compare to:



So we see that μ = 1/ξ. So by analogy we expect:



Or in other words,



If we use a ladder sum to calculate the density-density correlation function,



and extract D(q,ω), evidently we get the Drude result: D(q,ω) = D/(1-iωτ), where D = vF2τ/d and d = dimension I think. And,



What does this predict for localization? That there is none, since:



This would imply that the long time limit of the behavior of the density is:



And this is what we expect because in the classical file we solved for n(r) and found it goes as n(r) ~ exp(-r2/4Dt)/r, which in the long time limit goes as n(r) ~ 1/r. But in contrast, if we were to have found 1/ξ to be non-zero, then the density would go as n(r) ~ exp(-r/ξ)/r in the long time limit. This would indicate that the particle has basically stopped diffusing for r > ξ, as it is *exponentially* damped past r = ξ.